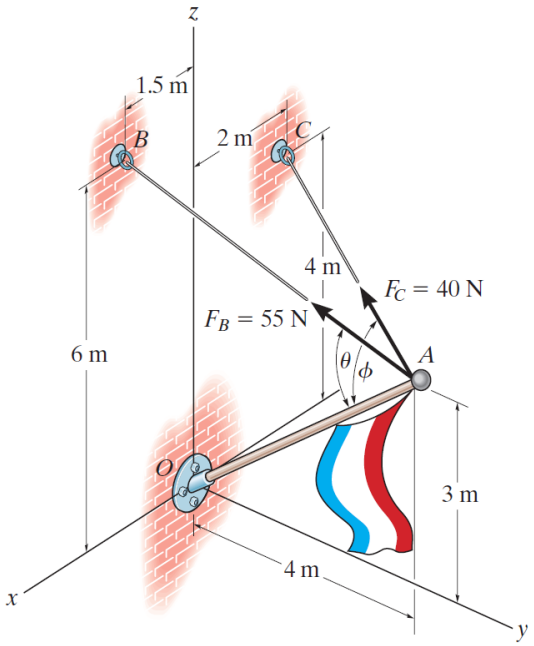
Problem 2-130

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



Prob. 2–130

Solution

Write the position vectors to the points O, A, B, and C.

$$\mathbf{r}_O = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_A = \langle 0, 4, 3 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 1.5, 0, 6 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -2, 0, 4 \rangle \text{ m}$$

The unit vector going from A to O is

$$\hat{\mathbf{u}}_{AO} = \frac{\mathbf{r}_O - \mathbf{r}_A}{|\mathbf{r}_O - \mathbf{r}_A|} = \frac{\langle 0, -4, -3 \rangle}{\sqrt{(0)^2 + (-4)^2 + (-3)^2}},$$

the unit vector going from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} = \frac{\langle 1.5, -4, 3 \rangle}{\sqrt{(1.5)^2 + (-4)^2 + (3)^2}},$$

and the unit vector going from A to C is

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} = \frac{\langle -2, -4, 1 \rangle}{\sqrt{(-2)^2 + (-4)^2 + (1)^2}}.$$

Take the dot product of $\hat{\mathbf{u}}_{AO}$ and $\hat{\mathbf{u}}_{AB}$ to determine θ.

$$\cos\theta = \hat{\mathbf{u}}_{AO} \cdot \hat{\mathbf{u}}_{AB} = \frac{\langle 0, -4, -3 \rangle}{\sqrt{(0)^2 + (-4)^2 + (-3)^2}} \cdot \frac{\langle 1.5, -4, 3 \rangle}{\sqrt{(1.5)^2 + (-4)^2 + (3)^2}} = \frac{14}{5\sqrt{109}}$$

$$\theta = \cos^{-1}\left(\frac{14}{5\sqrt{109}}\right) \approx 74.4^\circ$$

Take the dot product of $\hat{\mathbf{u}}_{AO}$ and $\hat{\mathbf{u}}_{AC}$ to determine ϕ.

$$\cos\phi = \hat{\mathbf{u}}_{AO} \cdot \hat{\mathbf{u}}_{AC} = \frac{\langle 0, -4, -3 \rangle}{\sqrt{(0)^2 + (-4)^2 + (-3)^2}} \cdot \frac{\langle -2, -4, 1 \rangle}{\sqrt{(-2)^2 + (-4)^2 + (1)^2}} = \frac{13}{5\sqrt{21}}$$

$$\phi = \cos^{-1}\left(\frac{13}{5\sqrt{21}}\right) \approx 55.4^\circ$$